

Bohr's complementarity relation and the violation of the \mathcal{CP} symmetry in high energy physics

Beatrix C. Hiesmayr and Marcus Huber

Faculty of Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria

We test Bohr's complementary relation, which captures the most counterintuitive difference of a classical and a quantum world, for single and bipartite neutral kaons. They present a system that is naturally interfering, oscillating and decaying. Moreover, kaons break the \mathcal{CP} symmetry (\mathcal{C} ...charge conjugation, \mathcal{P} ...parity). In detail we discuss the effect of the \mathcal{CP} violation on Bohr's relation, i.e. the effect on the "particle-like" information and the "wave-like" information. Further we show that the quantity that complements the single partite information for bipartite kaons is indeed concurrence, a measure of entanglement, strengthening our concept of entanglement. We find that the defined entanglement measure is independent of \mathcal{CP} violation while it has been shown that nonlocality is sensitive to \mathcal{CP} violation.

Keywords: Bohr's complementarity, entanglement, CP violation, neutral kaons

PACS: 03.67.Mn

I. INTRODUCTION

Bohr's complementarity principle or the closely related concept of duality in interferometric or double slit like devices are at the heart of quantum mechanics. The complementarity principle was phrased by Niels Bohr in an attempt to express the most fundamental difference between classical and quantum physics. According to this principle, and in sharp contrast to classical physics, in quantum physics we cannot capture all aspects of reality simultaneously, the information content obtained in one single setup is always limited. By choosing the setup, e.g. the double slit parameters, and thus the quantum state under investigation, the predictability, the *a priori* knowledge on the path taken, is simply calculated ("particle-like" information), whereas the contrast of the interference pattern ("wave-like" information) is observed by the experimenter. In the case of a pure state, the sum of the squares of these two quantities adds up to one, meaning that the whole available information, particle-like and wave-like, is conserved.

This principle has intensely been investigated both in theory and experiment mainly for photons, electrons and neutrons propagating through a double slit or through an interferometer. We here present a system which seems to be optimal for testing Bohr's complementary relation: It's a system which is 'created' by Nature already as an interfering system —thus there is no need for modeling by any experimenter— and are at the realm of an energy scale which are usually not used to test the fine workings of quantum physics. Moreover, it has a natural time evolution, oscillation and decay. In addition the system breaks a symmetry in high energy physics [1], the symmetry \mathcal{CP} (\mathcal{C} ...charge conjugation, \mathcal{P} ...parity), which is well studied, but lacks a deep understanding. Such a candidate is the neutral K-meson, shortly called kaon. One goal of this work is to discuss the effect of the symmetry violation to Bohr's complementarity principle.

We also go one step further and discuss an extension of

Bohr's idea to capture the information theoretic content of bipartite systems. For bipartite qubits [2] it has been shown that the missing part adding to the single properties of one partner of the pair is nothing else than concurrence. This is a computable function of entanglement of formation which is a measure of entanglement. We apply that to neutral kaons which is not straightforward, because, though neutral kaons are two-state systems, they have to be handled as an open quantum system due to their decay property [3, 4]. We demonstrate that this can be done in a useful and consistent way and it strengthens the result about concurrence presented in Ref. [4].

The work is organized as follows. We start by introducing Bohr's quantitative complementarity relation for single and bipartite qubits, then we give a short introduction into the quantum formalism of neutral kaons, followed by deriving both relations, single and bipartite, for neutral kaons and by discussing the effect of the symmetry violation.

II. BOHR'S QUANTITATIVE COMPLEMENTARITY RELATION

The qualitative well-known statement that "*the observation of an interference pattern and the acquisition of which-way information are mutually exclusive*" has been rephrased to a quantitative statement first by Greenberger and Yasin [5] and then refined by Englert [6]:

$$\mathcal{P}^2 + \mathcal{V}^2 \leq 1, \quad (1)$$

where the equality is valid for pure and the inequality for mixed quantum states. \mathcal{V} is the fringe visibility which quantifies the sharpness or contrast of the interference pattern ("the wave-like property"), whereas \mathcal{P} denotes the path predictability, i.e., the *a priori* knowledge one can have on the path taken by the interfering system ("the particle-like property"). In double slit experiment it is simply defined by $\mathcal{P} = |p_I - p_{II}|$, where p_I and p_{II} are the probabilities for taking each path ($p_I + p_{II} = 1$).

In the following we are not limiting to the double slit scenario, rather we write our interfering system generally in the computational basis

$$\tau = \frac{1}{2} \{ \mathbb{1}_2 + \vec{n} \cdot \vec{\sigma} \} \quad (2)$$

where the Bloch vector $\vec{n} \in \mathbb{R}^3$ and $|\vec{n}|^2 \leq 1$ and σ 's denote the Pauli matrices. Then the predictability and visibility can be expressed by

$$\mathcal{P} = |Tr(\sigma_z \tau)| = |n_z| \quad (3)$$

$$\mathcal{V} = |Tr(\sigma^+ \tau)| = |n_x + i n_y|, \quad (4)$$

and clearly Bohr's relation (1) holds. For all pure states, $|\vec{n}|^2 = 1$, no information is lost, for mixed states, $|\vec{n}|^2 \leq 1$, the loss is due to ignorance of individual particles, thus is a purely classical loss.

Note that here the \mathcal{V} is defined as the coherent superposition of the two orthogonal states, i.e. as coherence, which for qubits coincides with the visibility defined by the term that multiplies the interference term. This is not always the case, i.e. for neutral kaons, only if the Pauli matrices are chosen in the strangeness basis coherence coincides with visibility (see Section V).

One can make Bohr's complementary relation always exact by adding the quantity

$$M^2(\tau) = 2((Tr\tau)^2 - Tr(\tau^2)) \quad (5)$$

to the single particle property $\mathcal{S}^2 = \mathcal{P}^2 + \mathcal{V}^2$

$$\mathcal{S}^2(\tau) + M^2(\tau) = 1 \quad (6)$$

for all states (pure: $M(\tau) = 0$). $M(\tau)$ measures the mixedness or linear entropy which equals in this case the uncertainty of individual particles under investigation, clearly a "classical" uncertainty.

III. COMPLEMENTARITY OF BIPARTITE QUBITS AND CONCURRENCE

Let us now proceed to bipartite qubits in a state ρ . Considering one subsystem, clearly we have $\mathcal{S}_k^2 = \mathcal{P}_k^2 + \mathcal{V}_k^2 \leq 1$ where k denotes the chosen subsystem ($\mathcal{P}_k \equiv \mathcal{P}(\rho_k)$, $\mathcal{V}_k = \mathcal{V}(\rho_k)$ and $\rho_k = Tr_{-k}\rho$ is the partial trace over the other subsystem). As Jakob and Bergou [2] showed the following relation holds

$$\underbrace{\mathcal{P}_k^2 + \mathcal{V}_k^2}_{\text{one-particle property } \mathcal{S}} + \underbrace{\mathcal{C}^2}_{\text{two-particle property}} \leq 1 \quad (7)$$

where the equality sign is valid for all pure bipartite states. Thus the missing information adding to the single qubit information Alice or Bob is possessing, is a two-particle property and, surprisingly, nothing else than the concurrence, $\mathcal{C} \equiv \mathcal{C}(\rho)$, introduced by Hill and Wootters [7].

To compute concurrence one defines the flipped matrix $\tilde{\rho} = (\sigma_y \otimes \sigma_y) \rho^* (\sigma_y \otimes \sigma_y)$ where σ_y is the y -Pauli matrix and the complex conjugation is taken in computational basis. The concurrence is then given by the formula $\mathcal{C} = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$ where the λ_i 's are the square roots of the eigenvalues, in decreasing order, of the matrix $\rho \tilde{\rho}$.

Concurrence is a computable function of entanglement of formation, which is a measure of entanglement. It is nondecreasing under local operation and classical communication (LOCC) and does only depend on the density matrix and is consequently independent on any local basis. Entanglement of formation is defined by $\mathcal{EoF}(\rho) = \min_i \sum_i p_i S(Tr_l(|\psi_i\rangle\langle\psi_i|))$ where S is the von Neumann entropy $S(\rho) = -\rho \ln \rho$, the trace is taken over one subsystem (left or right) and ψ_i are the pure state decompositions of $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$ with $0 \leq p_i \leq 1$, $\sum p_i = 1$. There are certain criteria a measure should fulfill (see e.g. Ref. [8]) among them entanglement of formation is a good one. The outstanding problem of its additivity has recently been proven [9]. Concurrence is a computable function of this measure in the case of a bipartite system with two degrees of freedom, but it lacks additivity. The more interesting is it that concurrence is the missing two-particle information adding to the one-particle information, i.e. complementing the single particle information the experimenter Alice or Bob can obtain. Of course, concurrence has no operational meaning, but as shown in Ref. [10] for a restricted class of states quantum non-demolition tests of bipartite complementarity can be realized.

Clearly, for mixed states the complementarity relation is exact by exchanging the concurrence by $M(\rho_k)$. This mixedness obviously contains the entanglement property as well as classical uncertainty over individual particles and to separate these has so far only been achieved for bipartite qubits. However, for multi-particle or higher bipartite systems ideas to extend Bohr's principle have been discussed, Ref. [11].

Of course it remains an open question, whether the complementarity relation (7) is a universal physical feature of all quantum systems and in what respect. Therefore we move on to the neutral kaon system which is a two-state system, i.e. only two degrees of freedom can be measured, however, by decaying and breaking the \mathcal{CP} symmetry, differs from "normal" qubits.

IV. NEUTRAL KAONS AND THEIR TIME EVOLUTION

There have been a lot of puzzles about neutral kaons before a correct description was found. We here introduce only shortly the quantum mechanical formulation of these particles and work out the differences to qubits, which we need for analyzing Bohr's complementarity relation in the next section.

By the quantum number S , the strangeness, which is

conserved for strong interaction, we can distinguish between two states, the particle K^0 and the antiparticle \bar{K}^0 . Both strangeness states can decay via weak interaction into the same decay products, thus enables strangeness oscillation: if a K^0 is produced at time $t = 0$, one finds at a certain later time t a \bar{K}^0 . Thus neutral kaons oscillate between their particle and antiparticle state and have to be handled as a two-state system.

The time evolution is usually described via an effective Schrödinger equation which we write in the Liouville von Neumann form as

$$\frac{d}{dt} \tau_{ss} = -i H_{eff} \tau_{ss} + i \tau_{ss} H_{eff}^\dagger \quad (8)$$

where τ_{ss} is a 2×2 matrix and the Hamiltonian H_{eff} is non-Hermitian. Using the Wigner-Weisskopf approximation the effective Hamiltonian can be defined to be $H_{eff} = H - \frac{i}{2}\Gamma$ where the mass matrix H and the decay matrix Γ are both Hermitian and positive. This Wigner-Weisskopf approximation gives the exponential time evolution of the two diagonal states of H_{eff} :

$$\begin{aligned} |K_S(t)\rangle &= e^{-i\lambda_S t} |K_S\rangle, \\ |K_L(t)\rangle &= e^{-i\lambda_L t} |K_L\rangle, \end{aligned} \quad (9)$$

with $\lambda_{S/L} = m_{S/L} - i\frac{\Gamma_{S/L}}{2}$ where $m_{S/L}$ and $\Gamma_{S/L}$ are the masses and decay constants for the short/long-lived state $K_{S/L}$. What makes the neutral kaon systems so attractive for many physical analyzes, as e.g. considering Bell inequalities (e.g. Refs. [16, 17]) or quantum marking and eraser experiments [18, 19], is the huge factor between the two decay rates, i.e. $\Gamma_S \approx 600\Gamma_L$, and that the strangeness oscillation is $\Delta m = m_L - m_S \simeq \Gamma_S/2$.

A kaon or an antikaon is a superposition of the two mass-eigenstates K_S/K_L or the two \mathcal{CP} eigenstates denoted by K_1^0/K_2^0 :

$$\begin{aligned} |K^0\rangle &= \frac{N}{2p} \{ |K_S\rangle + |K_L\rangle \} = \frac{1}{\sqrt{2}} \{ |K_1^0\rangle + |K_2^0\rangle \} \\ |\bar{K}^0\rangle &= \frac{N}{2q} \{ -|K_S\rangle + |K_L\rangle \} = \frac{1}{\sqrt{2}} \{ -|K_1^0\rangle + |K_2^0\rangle \}. \end{aligned} \quad (10)$$

The weights $p = 1 + \varepsilon$, $q = 1 - \varepsilon$, with $N^2 = |p|^2 + |q|^2$ contain the complex \mathcal{CP} violating parameter ε which is measured to be $\varepsilon = (2.28 \pm 0.02) \times 10^{-3} e^{i\frac{\pi}{4}}$. It means that the short-lived K-meson decays dominantly into $K_S \rightarrow 2\pi$ with a lifetime $\tau_S = \Gamma_S^{-1}$ and the long-lived K-meson decays dominantly into $K_L \rightarrow 3\pi$ with a lifetime $\tau_L = \Gamma_L^{-1}$. However, due to \mathcal{CP} violation we observe a small amount $K_L \rightarrow 2\pi$. This introduces a small, but measurable difference between a world made of matter and a world made of antimatter.

Considering Eq. (9) we notice that the state is not normalized for $t > 0$. Indeed, we are not describing a system, for $t > 0$ a neutral kaon has a surviving and decaying component. Mathematically, the Hilbert space for single kaon evolving in time has to be divided into

a direct sum, i.e. $\mathbf{H}_{tot} = \mathbf{H}_s \oplus \mathbf{H}_f$ where s/f denotes “surviving” and “decaying” or “final” components.

There exist two approaches to view the kaonic system. One defines that the state for a short or long lived kaon after it propagates a certain time t is given by

$$|K_{S/L}(t)\rangle = \underbrace{e^{-i\lambda_{S/L}t} |K_{S/L}\rangle}_{\text{surviving}} + \underbrace{|\Omega_{S/L}(t)\rangle}_{\text{decaying}} \quad (11)$$

where the time evolution of the decaying components is obtained via

$$1 \stackrel{!}{=} \langle K_{S/L}(t) | K_{S/L}(t) \rangle = e^{-\Gamma_{S/L}t} + \langle \Omega_{S/L}(t) | \Omega_{S/L}(t) \rangle. \quad (12)$$

Differently stated, everything that is lost due to decay is added to the decaying components, such that the time evolution of the whole system is unitary. Mathematically, the Hilbert space is a direct sum and the “+” sign in the two above equations has to be understood in this way. The advantage working with this picture is that for initial pure states one deals only with the wave function formalism, the disadvantage is the time evolution of the decaying components cannot explicitly be given.

Another picture for kaons is given by an open quantum approach to particle decay [3, 4]: As time evolves the kaon interacts with an environment which causes the decay (in our case the environment plays the role as the QCD vacuum in quantum field theory, but has not to be modeled). In particular the time evolution of neutral kaons is described by a master equation [12, 13]

$$\frac{d}{dt} \tau = -i[\mathcal{H}, \tau] - \mathcal{D}[\tau] \quad (13)$$

where the dissipator under the assumption of complete positivity and Markovian dynamics has the well known general form $\mathcal{D}[\tau] = \frac{1}{2} \sum_j (\mathcal{A}_j^\dagger \mathcal{A}_j \tau + \tau \mathcal{A}_j^\dagger \mathcal{A}_j - 2\mathcal{A}_j \tau \mathcal{A}_j^\dagger)$. The density matrix τ lives on $\mathbf{H}_{tot} = \mathbf{H}_s \oplus \mathbf{H}_f$ and has the following decomposition

$$\tau = \begin{pmatrix} \tau_{ss} & \tau_{sf} \\ \tau_{sf}^\dagger & \tau_{ff} \end{pmatrix} \quad (14)$$

where τ_{ij} with $i, j = s, f$ denote 2×2 matrices. The Hamiltonian \mathcal{H} is the Hamiltonian H of the effective Hamiltonian H_{eff} extended to the total Hilbert space \mathbf{H}_{tot} and Γ of H_{eff} defines a Lindblad operator by $\Gamma = A^\dagger A$, i.e.

$$\mathcal{H} = \begin{pmatrix} H & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathcal{A} = \begin{pmatrix} 0 & 0 \\ A & 0 \end{pmatrix} \quad \text{with } A : \mathbf{H}_s \rightarrow \mathbf{H}_f.$$

Rewriting the master equation for τ , Eq. (14), on \mathbf{H}_{tot}

$$\dot{\tau}_{ss} = -i[H, \tau_{ss}] - \frac{1}{2} \{ A^\dagger A, \tau_{ss} \}, \quad (15)$$

$$\dot{\tau}_{sf} = -iH\tau_{sf} - \frac{1}{2} A^\dagger A \tau_{sf}, \quad (16)$$

$$\dot{\tau}_{ff} = A \tau_{ss} A^\dagger, \quad (17)$$

we notice that the master equation describes the original effective Schrödinger equation (8) but with properly normalized states, Ref. [3]. By construction the time evolution of τ_{ss} is independent of τ_{sf} , τ_{fs} and τ_{ff} . Further τ_{sf} , τ_{fs} completely decouples from τ_{ss} and thus can without loss of generality be chosen to be zero, they are not physical and can never be measured. With the initial con-

dition $\tau_{ff}(0) = 0$ the time evolution is *solely* determined by τ_{ss} —as expected for a spontaneous decay process—and formally given by integrating Eq. (17). It proves that the decay is Markovian and moreover completely positive.

Explicitly, the time evolution of a neutral kaon is given in the lifetime basis, $\{K_S, K_L\}$, by $(\tau_{SS} + \tau_{LL} = 1)$:

$$\tau(t) = \begin{pmatrix} e^{-\Gamma_S t} \tau_{SS} & e^{-i\Delta m t - \Gamma t} \tau_{SL} \\ e^{i\Delta m t - \Gamma t} \tau_{SL}^* & e^{-\Gamma_L t} \tau_{LL} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ (1 - e^{-\Gamma_L t}) \tau_{LL} & 0 \\ 0 & (1 - e^{-\Gamma_S t}) \tau_{SS} \end{pmatrix}. \quad (18)$$

V. THE QUANTITATIVE COMPLEMENTARITY FOR SINGLE KAONS

Let us define for an arbitrary state $\tau(t)$ of a neutral kaon evolving in time the following single partite property (t is suppressed)

$$\mathcal{S}^2(\tau) = \mathcal{P}^2(\tau) + \mathcal{V}^2(\tau) \quad (19)$$

where we define the predictability by

$$\mathcal{P}(\tau) = |\text{Tr}\left(\begin{pmatrix} \sigma_z & 0 \\ 0 & 0 \end{pmatrix} \tau\right)| = |\text{Tr}(\sigma_z \tau_{ss})| \quad (20)$$

and the coherence by

$$\mathcal{V}(\tau) = 2|\text{Tr}\left(\begin{pmatrix} \sigma^+ & 0 \\ 0 & 0 \end{pmatrix} \tau\right)| = 2|\text{Tr}(\sigma^+ \tau_{ss})|. \quad (21)$$

Obviously, only for the surviving components it makes sense to obtain “particle-like” or “wave-like” information.

Moreover, different to qubits we have two different physical options: the appearing Pauli matrices have to be defined relative to a basis choice. In the following we assume that τ_{ss} is given in the strangeness basis, $\{K^0, \bar{K}^0\}$, then the above Pauli matrices can be defined by “the strangeness choice”

$$\begin{aligned} S \equiv \sigma_z &:= |K^0\rangle\langle K^0| - |\bar{K}^0\rangle\langle \bar{K}^0| \\ \sigma^+ &:= |K^0\rangle\langle \bar{K}^0| \end{aligned} \quad (22)$$

or by “the \mathcal{CP} choice”

$$\begin{aligned} \mathcal{CP} \equiv \sigma_z &:= |K_1^0\rangle\langle K_1^0| - |K_2^0\rangle\langle K_2^0| \\ \sigma^+ &:= |K_1^0\rangle\langle K_2^0|. \end{aligned} \quad (23)$$

Let us in detail discuss these two options later.

Is \mathcal{S} a useful quantity?

For pure initial states τ we derive for any basis choice of the Pauli matrices

$$\mathcal{S}(\tau) = \mathcal{S}(\tau_{ss}) = \text{Tr}(\tau_{ss}) = \langle \psi(t) | \psi(t) \rangle, \quad (24)$$

thus one obtains the probability that a kaon survives until time t , i.e. the normalization to a surviving kaon $\text{Tr}(\tau_{ss}) \neq 1$ for $t > 0$. Or differently stated the result is the associated Bloch vector of the surviving block of τ , i.e. τ_{ss} . This is the same result as for qubits, however, the Bloch vector is not normalized due to the decay property. Clearly, \mathcal{S} is basis independent.

Therefore Bohr’s complementarity relation for initially pure kaon states reads

$$\mathcal{S}^2(\tau_{ss}) / (\text{Tr} \tau_{ss})^2 = 1 \quad (25)$$

which is analogous to the one for qubits, Eq. (1), except for the normalization due to decay. If we define the dynamical mixedness in a similar way as for qubits, Eq. (5), ranging from 0 to 1

$$M^2(\tau_{ss}) = 2((\text{Tr} \tau_{ss})^2 - \text{Tr}(\tau_{ss}^2)) \quad (26)$$

we obtain—in an analogous way to qubits, Eq. (6),—for a general single kaon state the following complementary relation for all states and times

$$\mathcal{S}^2(\tau_{ss}) / (\text{Tr} \tau_{ss})^2 + M^2(\tau_{ss}) / (\text{Tr} \tau_{ss})^2 = 1. \quad (27)$$

Note that the equation solely depends on the surviving property, i.e. τ_{ss} , and cannot be re-expressed by 4×4 density matrix τ describing the full quantum state under investigation. This is also seen, if one multiplies Eq.(27) by $\text{Tr}^2(\tau_{ss})$ and takes the squared root and uses $\text{Tr} \tau = 1 = \text{Tr} \tau_{ss} + \text{Tr} \tau_{ff}$, it turns into

$$\sqrt{\mathcal{S}^2(\tau_{ss}) + M^2(\tau_{ss})} + \text{Tr} \tau_{ff} = 1. \quad (28)$$

Now we can also recognize where the information flows. As the decoherence approach of particle decay only describes the flow from the surviving to the decaying states, $\text{Tr} \tau_{ff}$ is the missing term to the information about the surviving components. Note that this kind of information does not have the same dependence, i.e. is not quadratically, as the information loss due to classical uncertainty and this is also the reason why Bohr’s relation cannot be re-expressed solely with τ .

In summary, we have shown that the defined single partite property \mathcal{S} , Eq. (19), indeed captures the information obtainable from the quantum system under investigation, i.e. of a naturally interfering and decaying system, because

- (1) while predictability and coherence dependent on the basis choice, the single partite property \mathcal{S} is invariant to basis transformations,
- (2) for pure states \mathcal{S} equals the normalization to surviving kaons,
- (3) \mathcal{S} is complemented by the dynamical mixedness M , i.e. the the classical loss of obtainable information due to mixing.

And therefore Eq. (27) or Eq. (28) is the complementarity relation for a single kaonic quantum system we searched for, containing:

- (1) the obtainable information $S(\tau_{ss})$, contained in the surviving part of the propagating kaonic system
- (2) the classical uncertainty $M(\tau_{ss})$, quantifying the missing information about the surviving part of the propagating kaonic system
- (3) the information $Tr(\tau_{ff})$, that flows into the decay products.

Obviously, working with the surviving part normalized for all times by surviving kaons gives the identical Bohr's complementary relation, however, in the next section we show that doing that for bipartite kaons one runs into troubles.

How does \mathcal{CP} violation affect Bohr's complementary relation?

Clearly, the breaking of the \mathcal{CP} symmetry changes the physical states which are realized in Nature, Eq. (10), however, the derived Bohr's complementary relation, Eq. (27), works for all states of single kaons, consequently \mathcal{CP} violation does not invalidate Bohr's complementary relation. However, \mathcal{CP} violation shifts the kind of information from the predictability to the visibility and vice versa.

Let us explicitly discuss the following examples in the \mathcal{CP} choice (23), the time evolution of a K_S and of a K^0 . The predictability and the coherence of a K_S derives to:

$$\begin{aligned}\mathcal{P}(|K_S(t)\rangle\langle K_S(t)|) &= e^{-\Gamma_{st}} \frac{1 - |\varepsilon|^2}{1 + |\varepsilon|^2} \\ \mathcal{V}(|K_S(t)\rangle\langle K_S(t)|) &= e^{-\Gamma_{st}} \frac{2|\varepsilon|}{1 + |\varepsilon|^2}.\end{aligned}\quad (29)$$

Thus the a priori knowledge of being a \mathcal{CP} plus or minus state is decreased which results in a small coherence of the non-oscillating short lived state. In Fig. 1 the predictability \mathcal{P} and coherence \mathcal{V} normalized and not normalized to surviving components for a $|K^0(t)\rangle$ is drawn.

The strangeness choice (22) describes another physical situation and is discussed by the authors of Refs. [14]. They applied Bohr's complementarity relation in general to particle-antiparticle mixing systems, moreover to usual quantum systems as photons, neutron, ... but as well to Mott scattering experiments of identical particles or nuclei. They showed that all these two-state systems belonging to distinct fields of physics can be treated in a unified formalism, i.e. via the qualitative complementarity relation and moreover the effective number of visible fringes in an experiment can be derived.

Even for specific thermodynamical quantum system Bohr's complementary relation can be formulated and helps to understand in a simple information theoretic way the usually complex behaviour of such systems [15].

VI. QUANTITATIVE COMPLEMENTARITY FOR BIPARTITE KAONS AND CONCURRENCE

Two neutral kaons can also be produced in an entangled state, e.g. at the Φ -factory in Frascati kaon pairs are produced in the spin singlet state

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}} \{ |K^0\rangle \otimes |\bar{K}^0\rangle - |\bar{K}^0\rangle \otimes |K^0\rangle \} \quad (30)$$

which is obviously analogous to the one for qubits $|\psi\rangle = \frac{1}{\sqrt{2}} \{ |0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle \}$, however, the state for kaons evolves in time according to Wigner-Weisskopf approximation given in Eq. (9). In this section we analyze the Jakob-Bergou relation (7) for bipartite kaons.

In the previous section we have shown that for deriving the predictability and coherence only the surviving component is necessary. The surviving component of an arbitrary initially pure state at time $t=0$ can be parameterized in the $\{K_S, K_L\}$ basis in the following way

$$\begin{aligned}|\psi(t=0)\rangle &= \frac{1}{N(0)} \left\{ \right. \\ &\quad r_{SS}(0) |K_S\rangle \otimes |K_S\rangle + r_{SL}(0) |K_S\rangle \otimes |K_L\rangle \\ &\quad \left. + r_{LS}(0) |K_L\rangle \otimes |K_S\rangle + r_{LL}(0) |K_L\rangle \otimes |K_L\rangle \right\} \quad (31)\end{aligned}$$

where $N(0)$ is the normalization. The time evolution of the above surviving components are given by the Wigner-Weisskopf approximation (9), i.e.

$$\begin{aligned}r_{SS}(t) &= r_{SS} e^{i\phi_{SS}} \cdot e^{-i\lambda_{st}} e^{-i\lambda_{st}}, \\ r_{SL}(t) &= r_{SL} e^{i\phi_{SL}} \cdot e^{-i\lambda_{st}} e^{-i\lambda_{Lt}}, \\ r_{LS}(t) &= r_{LS} e^{i\phi_{LS}} \cdot e^{-i\lambda_{Lt}} e^{-i\lambda_{st}}, \\ r_{LL}(t) &= r_{LL} e^{i\phi_{LL}} \cdot e^{-i\lambda_{Lt}} e^{-i\lambda_{Lt}}.\end{aligned}\quad (32)$$

Therefore the time evolution of the surviving part of a

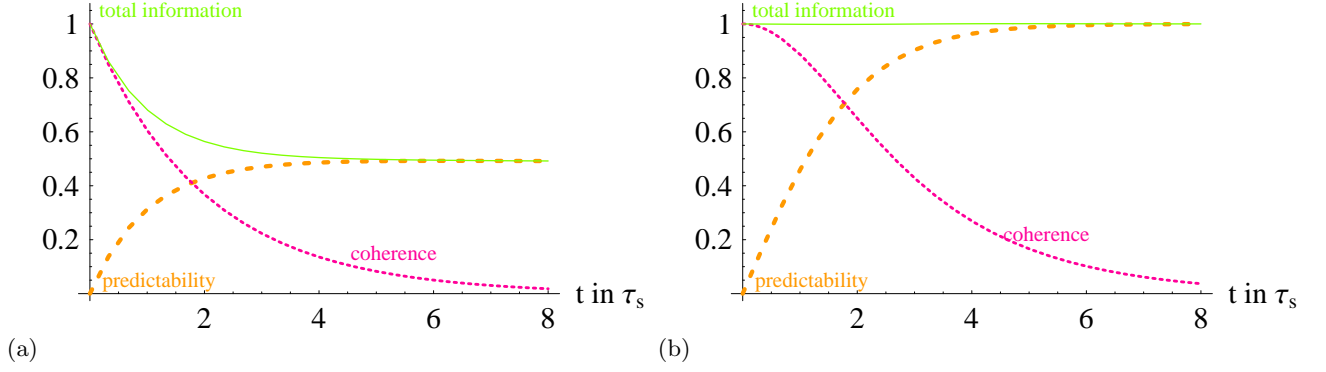


FIG. 1: The time evolution of the quantities, predictability \mathcal{P} , coherence \mathcal{V} and the total information $Tr(\tau_{ss})^2$ are drawn for the not normalized (a) and normalized (b) kaon state $|K^0\rangle$ in the \mathcal{CP} choice (23). At time $t = 0$ the K^0 is an equal superposition of the two \mathcal{CP} states, i.e. the predictability \mathcal{P} is zero, coherence \mathcal{V} is maximal. As time increases the short lived state dies out and one obtains more a priori knowledge on the \mathcal{CP} eigenstates. After about $4\tau_s$ the system is saturated.

general bipartite kaon state is given by

$$|\psi(t)\rangle = \frac{1}{N(0)} \left\{ \begin{aligned} &r_{SS}(t) |K_S\rangle \otimes |K_S\rangle + r_{SL}(t) |K_S\rangle \otimes |K_L\rangle \\ &+ r_{LS}(t) |K_L\rangle \otimes |K_S\rangle + r_{LL}(t) |K_L\rangle \otimes |K_L\rangle \end{aligned} \right\}. \quad (33)$$

Note that the above formula is not normalized for $t \geq 0$ (it is divided by $N(0)$ and not $N(t)$). Taking the partial trace gives the surviving part of the reduced matrix, i.e. $Tr_{-k}(|\psi(t)\rangle\langle\psi(t)|)$, representing the surviving part of the states of the kaon k , i.e. the one moving to the left hand side or to the right hand side.

Also we want that the following equation holds for arbitrary pure states as in the case for qubits (recall Eq. (7))

$$\frac{\mathcal{P}^2(Tr_{-k}(\rho_{ssss})) + \mathcal{V}^2(Tr_{-k}(\rho_{ssss})) + C^2}{Tr(\rho_{ssss}(t))^2} = 1 \quad (34)$$

where $\rho_{ssss}(t) = |\psi(t)\rangle\langle\psi(t)|$. In this case the quantity C has to equal

$$\begin{aligned} C &= (1 - \delta^2) \cdot \frac{2|r_{SS}(t)r_{LL}(t) - r_{SL}(t)r_{LS}(t)|}{N(0)^2} \\ &= (1 - \delta^2) \cdot e^{-2\Gamma t} \cdot \frac{2|r_{SS}r_{LL}e^{i(\phi_{SS}+\phi_{SS})} - r_{SL}r_{LS}e^{i(\phi_{SL}+\phi_{LS})}|}{N(0)^2}. \end{aligned} \quad (35)$$

Note that the time dependence factors out and damps C for any initial state in the same way with increasing time and C is in $[0, 1]$. **But is C the concurrence $C(\rho(t))$ for the states of bipartite kaons, a 16×16 density matrix $\rho(t)$ and, in turn, a measure of entanglement, as it is the case for qubits?**

THEOREM: 1 The quantity C , Eq. (35), equals the concurrence $\mathcal{C}(\rho_{ssss}(t))$ which is a computable function of entanglement of formation of a 16×16 density matrix $\rho(t) = \text{diag}\{\rho_{ssss}(t), \rho_{ssff}(t), \rho_{ffss}(t), \rho_{ffff}(t)\}$ describing bipartite kaons (derived in Ref. [4]). Here $\rho_{ssss}(t)$ equals $|\psi(t)\rangle\langle\psi(t)|$ defined in Eq. (33).

PROOF: 1 In Sect. IV we showed that single kaons can be handled by an open quantum system approach, in particular by a master equation of the Lindblad type. The density matrix $\tau(t)$ lives on $\mathbf{H}_{\text{tot}} = \mathbf{H}_s \oplus \mathbf{H}_f$ where s and f denote “surviving” and “decaying” or “final” components, and one finds for a single kaon evolving in time

$$\tau(t) = \begin{pmatrix} \tau_{ss}(t) & 0 \\ 0 & \tau_{ff}(t) \end{pmatrix} \quad (36)$$

where τ_{ss} is the 2×2 surviving block, i.e. is equivalent to the time evolved state without normalization, and τ_{ff} is 2×2 block which accounts for the decaying or final states.

Bipartite kaons are consequently described by a diagonal 16×16 density matrix,

$$\rho(t) = \text{diag}\{\rho_{ssss}(t), \rho_{ssff}(t), \rho_{ffss}(t), \rho_{ffff}(t)\}$$

where the first to forth blocks refer to both kaons survive, the kaon propagating to the left survives and the kaon propagating to the right decays, vice versa and both kaons decay. The block $\rho_{ssss}(t)$ is equivalent to the time evolved state Eq. (33). For the correctly normalized 16×16 state $\rho(t)$ entanglement of formation is defined. In Ref. [4] the author proves that it is equivalent to the computation of concurrence of ρ_{ssss} where the complex conjugation can be done in the $\{K^0, \bar{K}^0\}$ basis or in the $\{K_1, K_2\}$ basis by defining σ_y accordingly. Concurrence of ρ_{ssss} is indeed independent of any local basis choice and thus is the quantity we search for which completes the single particle property of bipartite kaons. (The proof that concurrence of ρ_{ssss} is equivalent to entanglement of formation of ρ holds also for mixed initial states, Ref. [4]. This

can also be seen by noting that ρ lives on $\mathbf{H}_{tot} \otimes \mathbf{H}_{tot}$, i.e. $\mathbf{H}_s \otimes \mathbf{H}_s \oplus \mathbf{H}_s \otimes \mathbf{H}_f \oplus \mathbf{H}_f \otimes \mathbf{H}_s \oplus \mathbf{H}_f \otimes \mathbf{H}_f$, and therefore, any decomposition of ρ divides into a decomposition in each subspace. In the minimizing function of entanglement of formation only the part in $\mathbf{H}_s \otimes \mathbf{H}_s$ can contribute because there is no entanglement between surviving and decaying parts.)

Does it make sense to work with states normalized to surviving kaons?

This could prove to be quite misleading as the concurrence could rise through the decay property. For example consider the maximally entangled spin singlet state superposed with a fast decaying state as e.g. $|K_S\rangle \otimes |K_S\rangle$, see also Fig. 2 (b). Then the probability of finding a more entangled state actually does increase with time, because normalizing to surviving kaons increases the percentage of the entangled part. This can be understood for a single pair, however, not in the ensemble case. If one observes for a single pair until a certain time t no decay, then the knowledge about this pair increased, the information of the environment “no decay observed” was read out. This is what is done by normalizing to surviving pairs or differently stated it is a state preparation. In the ensemble case the experimenter obtains the probabilities or relative frequencies by dividing by the number of all pairs produced at time $t = 0$, not reading out the environment, but taking the information into account which flows into the environment through the decay property, in very same way as decoherence is described.

Thus the quantities of Fig. 2 (a) represent the different information content of the system evolving in time, whereas Fig. 2 (b) represent the different information content obtained at each time point for a different state, i.e. different experimental setups.

But there is another interesting fact we learn. In the case of single kaons we have noticed that due to \mathcal{CP} violation the quantities \mathcal{P} and \mathcal{V} change, however, \mathcal{S} was not affected.

Is concurrence, a measure of entanglement, also affected by \mathcal{CP} violation?

The maximally entangled antisymmetric Bell states which is e.g. produced in the e^+e^- machine DAΦNE is given by

$$\begin{aligned} |\psi^-\rangle_{K^0, \bar{K}^0} &:= \frac{1}{\sqrt{2}} \{ |K^0 \bar{K}^0\rangle - |\bar{K}^0 K^0\rangle \} = \\ \frac{1}{(1-\delta^2)} |\psi^-\rangle_{K_S, K_L} &:= \frac{1}{\sqrt{2}(1-\delta^2)} \{ |K_S K_L\rangle - |K_L K_S\rangle \} \end{aligned} \quad (37)$$

The remaining maximally entangled states are given by

$$\begin{aligned} |\psi^+\rangle_{K^0, \bar{K}^0} &= \frac{1}{(1-\delta^2)} (-) |\phi^-\rangle_{K_S, K_L}, \\ |\phi^+\rangle_{K^0, \bar{K}^0} &= \frac{1}{(1-\delta^2)} \{ |\phi^+\rangle_{K_S, K_L} + \delta |\psi^+\rangle_{K_S, K_L} \}, \\ |\phi^-\rangle_{K^0, \bar{K}^0} &= \frac{1}{(1-\delta^2)} \{ \delta |\phi^+\rangle_{K_S, K_L} + |\psi^+\rangle_{K_S, K_L} \}. \end{aligned} \quad (38)$$

The concurrence, Eq. (35), derives for all maximally entangled states to 1 (the above set of states are of course not the only set of maximally entangled states, but clearly the concurrence is still maximal). This is also seen if Eq. (35) is rewritten as

$$\mathcal{C}(\rho_{ssss}(t)) = \mathcal{C}(\rho_{ssss}(0)) \cdot (1-\delta^2) \cdot e^{-2\Gamma t}. \quad (39)$$

The term $1-\delta^2$ compensates the ones of wave function in the lifetime basis, Eq. (37) and Eq. (38), therefore concurrence is independent of \mathcal{CP} violation. Moreover, note that the time evolution factors out and the entanglement of the system at time $t = 0$ decreases for all states in the very same way, i.e. entanglement decreases due to the decay property.

In summary, \mathcal{CP} violation “deforms” the state space of single kaons because the time evolving states are not identical to the \mathcal{CP} eigenstates and this shifts the information from \mathcal{P} to \mathcal{V} or vice versa, but the state space of bipartite system is not “deformed”. This means that the two-particle property, concurrence, is not affected directly by \mathcal{CP} violation. The more interesting is the result of Ref. [16] where the authors show that any non-zero δ leads to a violation of a Bell inequality, thus clearly testing a two-particle property, namely nonlocality.

VII. CONCLUSIONS

We have shown that applying the quantum mechanical formalism to describe a naturally interfering decaying and \mathcal{CP} violating system in particle physics delivers results consistent with our expectation. Bohr’s complementarity seems to be an intrinsic feature of our physical reality and in addition it fits very well with our understanding of entanglement.

In detail we show that Bohr’s quantitative relation can be obtained by only operating on the surviving part of the density matrix describing neutral kaons. Also we have demonstrated that the amount of information decreasing through decay is equal to the probability of obtaining a decay product, which validates the view of the kaonic system as an open quantum system.

In the case of bipartite kaonic systems we are able to prove that the amount of information, that should be found in an entanglement measure according to the complementarity relation, is indeed equal to the previously defined concurrence of the whole kaonic system.

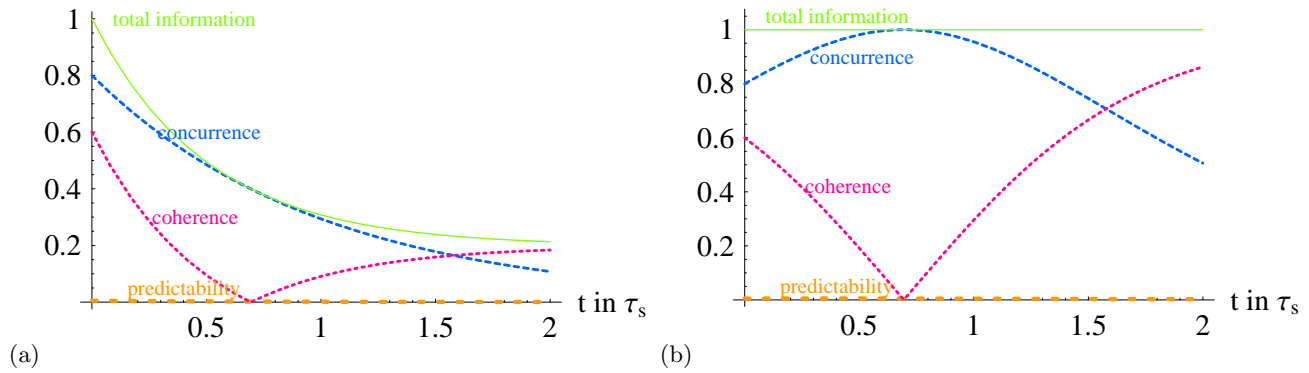


FIG. 2: Predictability \mathcal{P} , coherence \mathcal{V} , concurrence C and the total information $Tr(\rho_{ssss})^2$ are drawn for the not normalized (a) and normalized (b) state $|\psi\rangle = \frac{1}{\sqrt{2}}(|\psi^-\rangle_{K^0, \bar{K}^0} + |K_S\rangle \otimes |K_S\rangle)$ evolving in time. Here one can clearly see the effects of normalizing to surviving pairs, concurrence rises with time. This is due to the fact that normalizing to surviving pairs will express the percentage contribution of the different point quantities to the total information and captures the information content of a pair that survived until t , i.e. for every time point t a different state is discussed.

Also new light has been shed upon the violation of a symmetry in particle physics, by showing that it shifts obtainable information about our reality to different aspects, without violating the complementarity principle, i.e. from predictability \mathcal{P} to coherence \mathcal{V} and vice versa.

It is also interesting that the single partite property $S = \sqrt{\mathcal{P}^2 + \mathcal{V}^2}$ and the entanglement measure C , a bipartite property, are unchanged by \mathcal{CP} violation, whereas in Refs. [16, 17] it is shown that any nonzero amount of \mathcal{CP} violation leads to a violation of Bell' inequality, i.e. testing another two-particle property, namely nonlocality. This strengthens also the view that nonlocality and Bohr's complementing quantity, entanglement, are different quantum features. For qudits this view was suggested

in Ref. [20] and for kaons in Ref. [4].

Demonstrating the working of Bohr's relation for kaons also helps to clarify the "old" discussions on usual double slit scenarios, e.g. Ref. [21, 22]. Moreover, by their time evolution the "which width" information corresponding to "which way" information is changed automatically in time opening options of quantum marking and erasure experiments [18]. Because of different measurement procedures, a special feature of kaons, the very working of a quantum eraser can in a novel way be demonstrated [19] and an experimental realization at the DAΦNE machine (Frascati, Rom) is under investigation.

Acknowledgement: Many thanks to R.A. Bertlmann and G. Garbarino for enlightening discussions.

-
- [1] J.H. Christenson, J.W. Cronin, V.L. Fitch and R. Turlay, Phys. Rev. Lett. **13**, 138 (1964).
 - [2] M. Jakob and J.A. Bergou, "Quantitative complementarity relations in bipartite systems", arXiv: quant-ph/0302075v1.
 - [3] R.A. Bertlmann, W. Grimus and B.C. Hiesmayr, Phys. Rev. A **73**, 054101 (2006).
 - [4] B.C. Hiesmayr, Eur. Phys. J. C **50**, 73 (2007).
 - [5] D. Greenberger and A. Yasin, Phys. Lett. A **128**, 391 (1988).
 - [6] B.-G. Englert, Rev. Lett. **77**, 2154 (1996).
 - [7] S. Hill and W.K. Wootters, Phys. Rev. Lett. **78**, 5022 (1997).
 - [8] D. Bruss, J. Math. Phys. **43**, 4237 (2002).
 - [9] G.A. Paz-Silva and J.H. Reina, "Full Additivity of the Entanglement of Formation", arXiv: 0704.1180.
 - [10] F. de Melo, S.P. Walborn, J.A. Bergou and L. Davidovich, "Quantum non-demolition test of bipartite complementarity", arXiv: quant-ph/0702079.
 - [11] T.E. Tessier, Found. Phys. Lett. **18**(2), 0403022 (2005).
 - [12] G. Lindblad, Comm. Math. Phys. **48**, 119 (1976).
 - [13] V. Gorini, A. Kossakowski, and E.C.G. Sudarshan, J. Math. Phys. **17**, 821 (1976).
 - [14] A. Bramon, G. Garbarino and B.C. Hiesmayr, Phys. Rev. A **69**, 022112 (2004).
 - [15] B.C. Hiesmayr and V. Vedral, "Interferometric wave-particle duality for thermodynamical systems", arXiv: quant-ph/0501015.
 - [16] R.A. Bertlmann, W. Grimus and B.C. Hiesmayr, Phys. Lett. A **289**, 21 (2001).
 - [17] R.A. Bertlmann and B.C. Hiesmayr, Phys. Rev. A **63**, 062112 (2001).
 - [18] A. Bramon, G. Garbarino and B. C. Hiesmayr, Phys. Rev. Lett. **92**, 020405 (2004).
 - [19] A. Bramon, G. Garbarino and B. C. Hiesmayr, Phys. Rev. A **68**, 062111 (2004).
 - [20] A. Acin, R. Gill and N. Gisin, Phys. Rev. Lett. **95**, 210402 (2005).
 - [21] A. Luis and L.L. Sanchez-Soto, Phys. Rev. Lett. **81**, 4031 (1998).
 - [22] B.-G. Englert, M. O. Scully and H. Walther, J. of Modern Optics **47**, 2213 (2000).